

α_s from the Lattice and Hadronic τ Decays

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Abstract. Until recently, determinations of $\alpha_s(M_Z)$ from hadronic τ decays and the analysis of short-distance-sensitive lattice observables yielded results which, though precise, were not in good agreement. I review new analyses that bring these into good agreement and provide some details on the source of the main changes in the τ decay analysis.

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Until recently the determinations of $\alpha_s(M_Z)$ from (i) perturbative analyses of short-distance-sensitive lattice observables (yielding $0.1170(12)$ [1]), and (ii) from finite energy sum rule (FESR) analyses involving hadronic τ decay data (yielding $0.1212(11)$ [2]), both claiming high precision, produced central values differing from one another by $\sim 3\sigma$. In the past year, significant updates to both analyses have appeared, bringing the two determinations into excellent agreement. We outline the important features of these updates responsible for this change in what follows.

The original lattice determination [1], employed a number of lattice observables, O_k , and the perturbative $D = 0$ expansions for these observables,

$$O_k = \sum_{N=1} \bar{c}_N^{(k)} [\alpha_V(Q_k)]^N \equiv D_k \alpha_V(Q_k) \sum_{M=0} c_M^{(k)} [\alpha_V(Q_k)]^M \quad (1)$$

where α_V is a coupling defined in Refs. [1, 3], $Q_k = d_k/a$ is the BLM scale for O_k , and the $\bar{c}_{1,2,3}^{(k)}$ (equivalently, $D_k, c_1^{(k)}, c_2^{(k)}$) relevant to the MILC lattice data employed have been computed in 3-loop lattice perturbation theory and, with the corresponding d_k , compiled in Refs. [1, 3]. m_q -dependent contributions were removed by extrapolation, using data, and non-perturbative (NP) m_q -independent higher D contributions treated as being dominated by $D = 4$ gluon condensate terms, which were fitted and removed independently for each O_k . Data with lattice spacings $a \sim 0.18, 0.12$, and 0.09 fm were employed. At these scales it was necessary to fit at least one additional coefficient in Eq. (1) [1]. More recently, new MILC ensembles with $a \sim 0.15$ and 0.06 fm became available and were incorporated into the updated analyses of Refs. [3, 4]. One very new $a \sim 0.045 \text{ fm}$ ensemble was also employed in [3]. The new analyses thus involve data whose range of scales is greater and whose highest scale is larger (and hence more perturbatively-dominated). The two re-analyses, moreover, differ somewhat in their strategies, allowing for useful cross-checks. First, the two analyses employ a different choice of coupling, that of Ref. [3] leaving residual perturbative uncertainties in the conversion from the V to \overline{MS} scheme, that of Ref. [4] leaving them in the effects of the truncated β function, which can be suppressed by focussing on finer lattices [4].

TABLE 1. $\alpha_s(M_Z)$ and the shift $\delta_{D=4}$ induced by the $D = 4$ m_q -independent NP correction with charmonium sum rule input for $\langle \alpha_s G^2 \rangle$

O_k	$\alpha_s(M_Z)$ (HPQCD)	$\alpha_s(M_Z)$ (CSSM)	$\delta_{D=4}$
$\log(W_{11})$	0.1185(8)	0.1190(11)	0.7%
$\log(W_{12})$	0.1185(8)	0.1191(11)	2.0%
$\log(W_{12}/u_0^6)$	0.1183(7)	0.1191(11)	5.2%
$\log(W_{11}W_{22}/W_{12}^2)$	0.1185(9)	N/A	32%
$\log(W_{23}/u_0^{10})$	0.1176(9)	N/A	53%
$\log(W_{14}/W_{23})$	0.1171(11)	N/A	79%
$\log(W_{11}W_{23}/W_{12}W_{13})$	0.1174(9)	N/A	92%

Second, Ref. [3] performs an improved treatment of m_q -independent NP contributions, fitting a range of $D \geq 4$ forms to data, while Ref. [4] restricts its attention to observables where the corresponding $D = 4$ contributions, estimated using charmonium sum-rule input for $\langle \alpha_s G^2 \rangle$ [5], can be shown to be small. Even with finer lattice scales, at least one additional coefficient in Eq. (1) must be fit. The resulting fitted α_s provide an excellent representation of the scale dependence of the O_k . The results of the two re-analyses are in good agreement, and differ by only $\sim 1\sigma$ from the results of [1]. The results, run to the $n_f = 5$ scale M_Z , are shown in Table 1 for the three most perturbative and four least perturbative of the O_k studied in [3]. W_{kl} is the $k \times l$ Wilson loop and $u_0 = W_{11}^{1/4}$. Also shown is a measure, $\delta_{D=4}$, of the expected importance of m_q -independent NP contributions to O_k , relative to the $D = 0$ contribution of interest in the determination of α_s . $\delta_{D=4}$ is the percent shift in the scale dependence between $a \sim 0.12$ and $a \sim 0.06$ fm resulting from first computing O_k using raw simulation values for the relevant W_{kl} , and then re-computing it after subtracting the known leading order m_q -independent $D = 4$ contributions, estimated using charmonium sum rule input for $\langle \alpha_s G^2 \rangle$ [5]. Sizable NP effects are thus expected for the O_k in the lower half of the table. The fact that, after such large contributions are approximately fitted and removed, the resulting $\alpha_s(M_Z)$ are in such good agreement with those obtained by analyzing more $D = 0$ -dominated O_k argues strongly for the reliability of the approach and gives even higher confidence in results based on the most UV-sensitive of the O_k , $\log(W_{11})$, where the estimated $D = 4$ subtraction is very small, producing a shift of only 0.0001 in $\alpha_s(M_Z)$ [4].

In the SM, with $\Gamma_{V/A;ud}^{had}$ the τ width to hadrons through the $I = 1$ V or A current, Γ_e the τ electronic width, $y_\tau = s/m_\tau^2$, and S_{EW} a known short-distance EW correction, $R_{V/A;ud} = \Gamma_{V/A;ud}^{had}/\Gamma_e$ is related to the spectral functions $\rho_{V/A;ud}^{(J)}(s)$ of the spin J scalar correlators, $\Pi_{V/A;ud}^{(J)}(s)$, of the V/A current-current two-point functions by [8]

$$dR_{V/A;ud}/dy_\tau = 12\pi^2 S_{EW} |V_{ud}|^2 \left[w_{00}(y_\tau) \rho_{V/A;ud}^{(0+1)}(s) - w_L(y_\tau) \rho_{V/A;ud}^{(0)}(s) \right], \quad (2)$$

where $w_{00}(y) = (1-y)^2(1+2y)$, $w_L(y) = 2y(1-y)^2$ and, up to $O[(m_d \pm m_u)^2]$ corrections, $\rho_{V;ud}^{(0)}(s) = 0$ and $\rho_{A;ud}^{(0)}(s) = 2f_\pi^2 \delta(s - m_\pi^2)$. $\rho_{V/A;ud}^{(0+1)}(s)$ is thus accessible from

experimental results for $dR_{V/A;ud}/dy_\tau$ [6, 7]. The corresponding correlator combination satisfies, for any s_0 and any analytic $w(s)$, the FESR relation

$$\int_0^{s_0} w(s) \rho_{V/A;ud}^{(0+1)}(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Pi_{V/A;ud}^{(0+1)}(s) ds , \quad (3)$$

where the OPE can be employed on the RHS for large enough s_0 . For typical weights $w(s)$ and s_0 above $\sim 2 \text{ GeV}^2$, $\left[\Pi_{V/A;ud}^{(0+1)} \right]_{OPE}$ is strongly $D = 0$ dominated, hence largely determined by α_s . Use of polynomial weights, $w(y)$, with $y = s/s_0$, helps in quantifying higher D contributions, most of which must be fit to data, since (with N the degree of $w(y)$), (i) up to corrections of $O(\alpha_s^2)$, the OPE series terminates at $D = 2N + 2$, and (ii) integrated OPE contributions with $D = 2k + 2$ scale as $1/s_0^k$, allowing contributions with different D to be separated via their differing s_0 -dependences.

Earlier τ decay determinations were based on combined analyses of the $s_0 = m_\tau^2$, $km = 00, 10, 11, 12, 13$, $w_{km}(y) = w_{00}(y)(1-y)^k y^m$ “spectral weight FESRs” (see, e.g., Refs. [2, 6]). The most recent versions [2, 9] employ the 5-loop $D = 0$ V/A Adler function result [9], as do subsequent studies [10, 11, 12, 13, 14]. The w_{km} analysis relies crucially on the additional non-trivial assumption that $D = 10, \dots, 16$ contributions, each in principle present for one or more of the w_{kl} employed, can, in all cases, be safely neglected, an assumption of potential relevance since a $\sim 1\%$ determination of $\alpha_s(M_Z)$ requires control of $D > 4$ NP contributions to $\lesssim 0.5\%$ of the leading $D = 0$ term. Tests of this assumption were performed in Ref. [12] by (i) studying the match between the OPE integrals, evaluated using fitted OPE parameters, and the corresponding experimental weighted spectral integrals as a function of s_0 , and (ii) using the same data and fitted OPE parameters as input to FESRs for different $w(y)$ involving the same set of OPE parameters. It was found that, in the window $\sim 2 \text{ GeV}^2 < s_0 \leq m_\tau^2$, the match between the optimized OPE integrals and experimental spectral integrals generated by the ALEPH data and fits is typically poor, not just for the w_{kl} employed in the ALEPH analysis, but also for other degree ≤ 3 weights, which depend only on the $D = 0, 4, 6, 8$ OPE parameters included in the ALEPH fit. Similar problems, albeit with somewhat reduced OPE-spectral integral discrepancies, are also found for the OPAL data and fit parameter set. Refs. [12] also performed analyses based on alternate weights, $w_N(y) = 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$, designed to suppress $D = 2N + 2$ contributions relative to the leading $D = 0$ terms, and hence optimize the determination of α_s . It was found that (i) the fits for α_s obtained using different $w_N(y)$, and also analyzing separately the V, A and V+A channels, are all in excellent agreement; (ii) the impact of the $D > 4$ OPE contributions is, as intended, small; and (iii) unlike the situation found using the ALEPH and OPAL fits, the w_N FESR fit parameter set produces OPE spectral integral results which match the corresponding spectral integrals within experimental errors for other degree ≤ 3 weights (including the kinematic weight w_{00}) over the whole of the s_0 window noted above. One should bear in mind that, in terms of its size relative to the crucial $D = 0$ term, it is a factor of between 7 and 814 times safer to neglect $D > 8$ contributions in the w_N analyses than it was in the higher w_{kl} FESRs of the ALEPH and OPAL analyses [12]. In view of the fact that (i) the older analyses, which should produce results in agreement with those of the corresponding w_N analyses when using the same data, instead produce

significantly larger α_s , and (ii) the results of the old analyses, considered at lower s_0 , produce optimized OPE integrals not in agreement within errors with the corresponding experimental spectral integrals, and, moreover, significantly inferior to the matches obtained using the $\{w_N\}$ analysis fit parameters (see, e.g., the Figures in Refs. [12]), it seems clear that the results of the $\{w_N\}$ analysis should be taken to supercede those of the earlier combined w_{kl} analyses. The favored τ decay result for α_s is thus

$$\alpha_s(M_Z) = 0.1187(16) , \quad (4)$$

in excellent agreement with the lattice determination.

We conclude with a few comments on other recent results for α_s from hadronic τ decays. First, note that Refs. [9, 2, 14] employ as input for their $D = 6, 8$ OPE parameter values, the results obtained in either the 2005 or 2008 ALEPH combined $s_0 = m_\tau^2$ spectral weight FESR analysis. They thus lead to s_0 -dependent OPE integrals which do not match the corresponding spectral integrals within experimental errors, and whose matches are inferior to those produced by the OPE parameters obtained from the w_N FESR analyses. Ref. [11] (whose results also lead to an OPE-spectral integral mismatch [12], this time resulting from the use of a different set of assumed values for the required $D = 6, 8$ input [12]) however, raises an interesting question about the relative reliability of the FOPT and CIPT prescriptions for evaluating the truncated $D = 0$ series, one in need of, and undergoing, further investigation. Also relevant in this regard is the observation of Ref. [14], which shows a larger-than-previous-anticipated FOPT uncertainty associated with the dependence of the truncated FOPT result on the point on the OPE contour chosen as the fixed scale.

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